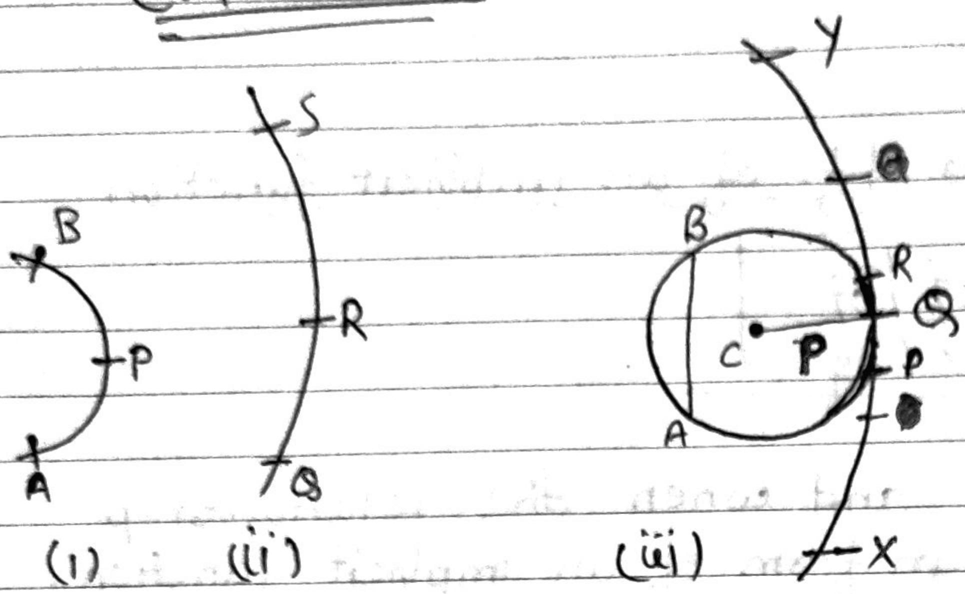
Zoom Class 16/04/2020

Curvature



Curvature is said to be the amount of bend

at a given point in any curve. From figure (i) and (ii) we can infer that the curvature at point P in curve AB is greater than the point R on curve QS.

There are different measures of curvature as follows and are represented in figure (iii)

(1) Circle of Curvature

Let Q be any point on the curve XY and two points P and R are very near to it. When P and R tends towards point Q there is a limiting position to form a circle passing through all three points. This limiting point of circle is called circle of curvature. shown in fig (iii)

(ii) Centre of Curvation.

The centre 'C' of circle of curvature is called centre of curvature

(iii) Radius of Curvature

The radius 'ρ' of circle of curvature is called radius of curvature (ρ) {rho}.

(iv) Chord of Curvature

Any chord AB drawn inside the circle of curvature is called chord of curvature.

Intrinsic Formula for radius of Curvature

If the Eqn of curve is given in terms of s and ψ

where ψ is the angle made by the tangent on x-axis

and s is any arcual distance made at a fixed point.

then

$$\boxed{\rho = \frac{ds}{d\psi}}$$

Cartesian formula for 'ρ':

If eqn of curve is $y = f(x)$

then $\frac{dy}{dx} = \tan\psi$ —— (1)

Diffrentiating (1) w.r.t. 'x' we get

$$\sec^2\psi \frac{d\psi}{dx} = \frac{d^2y}{dx^2}$$

$$\Rightarrow \sec^2\psi \frac{d\psi}{ds} \cdot \frac{ds}{dx} = \frac{d^2y}{dx^2}$$

$$\Rightarrow \sec^2\psi \cdot \frac{1}{\rho} \cdot \frac{1}{\cos\psi} = \frac{d^2y}{dx^2} \quad \left\{ \because \frac{ds}{dx} = \frac{1}{\cos\psi} \right\}$$

$$\Rightarrow \sec^3\psi \cdot \frac{1}{\rho} = \frac{d^2y}{dx^2}$$

$$\Rightarrow \frac{1}{\rho}(1+\tan^2\psi)^{3/2} = \frac{d^2y}{dx^2}$$

$$\Rightarrow \frac{1}{\rho}\left(1+\left(\frac{dy}{dx}\right)^2\right)^{3/2} = \frac{d^2y}{dx^2}$$

$$\Rightarrow \boxed{\rho = \frac{\left\{1+(dy/dx)^2\right\}^{3/2}}{(d^2y/dx^2)}}$$

where $\frac{d^2y}{dx^2} \neq 0$.

<u>Note</u> : Cartesian formula will fail if tangent is parallel to y-axis i.e., $\psi = 90°$
In this case

$$\rho = \frac{\left\{1+\left(\frac{dx}{dy}\right)^2\right\}^{3/2}}{\left(\frac{d^2x}{dy^2}\right)} \quad \text{where } \frac{d^2x}{dy^2} \neq 0$$

Parametric formula

If eqn of curve is $x = f(t)$, $y = g(t)$

then $$\boxed{\rho = \frac{(x'^2 + y'^2)^{3/2}}{x'y'' - y'x''}}$$

where $x' = f'(t)$, $x'' = f''(t)$
$y' = g'(t)$, $y'' = g''(t)$

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Ques Find radius of Curvature for the curve $s = c\tan\psi$ at point (s, ψ)

Soln $\because$ It is case of (s, ψ) we use intrinsic formula

$$\rho = \frac{ds}{d\psi}$$

$\because s = c\tan\psi$
$\Rightarrow ds = c\sec^2\psi \, d\psi$

$\Rightarrow \frac{ds}{d\psi} = c\sec^2\psi \Rightarrow \rho = c\sec^2\psi$

Ans